**Exam. Code : 211003**
**Subject Code : 3853**

## M.Sc. Mathematics 3rd Semester

## MATI' 578 : OPERATIONS RESEARCH—I

Time Allowed—3 Hours] [Maximum Marks—100

**Note :—** Attempt **ten** questions in all, by selecting at least **two** questions from each Unit. All questions carry equal marks.

### UNIT—I

1. Let an LPP have a basic feasible solution. If we drop one of the basis vectors and introduce a non-basis vector in the basis set then show that the new solution obtained is also a basic feasible solution.

2. A company has three opreational departments (weaving, processing and packing) with a capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per meter respectively. One meter of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one metre of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One metre of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively . Formulate the linear programming problem to find the product mix to maximize the profit.

3. Use Penality method to :

   Maximize $Z = 6x_1 + 4x_2$

   subject to the Constraints :

   $2x_1 + 3x_2 \le 30,$

   $3x_1 + 2x_2 \le 24,$

   $x_1 + x_2 \ge 3,$

   $x_1 \ge 0$ and $x_2 \ge 0.$

4. Use two-phase simplex method to :

   Maximize $Z = 5x_1 + 8x_2$

   subject to the Constraints :

   $3x_1 + 2x_2 \ge 3,$

   $x_1 + 4x_2 \ge 4,$

   $x_1 + x_2 \le 5,$

   $x_1, x_2 \ge 0.$

## UNIT—II

5. Let a primal problem be

   Maximize $f(x) = CX$

   subject to $AX \le b, \ X \ge 0, X^T, C \in R^n$

   and the associated dual be

   Minimize $g(W) = b^T W$

   subject to $A^T W \ge C^T, W \ge 0, W^T, b^T \in R^m$

   If $X_o(W_o)$ is an optimum solution to the primal (dual) then show that there exists a feasible solution $W_o(X_o)$ to the dual (primal) such that $CX_o = b^T W_o$.

6. State and prove complementary theorem.

7. Use duality to solve the following LPP :—

Maximize $Z = x_1 - x_2 + 3x_3 + 2x_4$

subject to

$x_1 + x_2 \geq -1,$

$x_1 - 3x_2 - x_3 \leq 7,$

$x_1 + x_3 - 3x_4 = -2;$

$x_1, x_2, x_3, x_4 \geq 0.$

8. Use dual simplex method to :

Minimize $Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

subject to the Constraints :

$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12,$

$x_2 + 5x_3 - 6x_4 \geq 10,$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

$x_1, x_2, x_3, x_4 \geq 0.$

## UNIT—III

9. Explain transportation problem and show that it can be considered as an LPP.

10. How the problem of degeneracy arises in a transportation problem ? Explain how does one overcome it.

11. Explain the Least Cost method for finding out an initial basic feasible solution of transportation problem.

12. If the matrix elements represents the unit transportation times, solve the following transportation problem :

To

| From | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Available |
|---|---|---|---|---|---|
| $O_1$ | 10 | 0 | 20 | 11 | 15 |
| $O_2$ | 1 | 7 | 9 | 20 | 25 |
| $O_3$ | 12 | 14 | 16 | 18 | 5 |
| Required | 12 | 8 | 15 | 10 | 45 |

## UNIT—IV

13. Discuss the various steps in finding the solution of an assignment problem.

14. A Car hire company has one car at each of five depots a, b, c, d end e. A customer in each of the five towns A, B, C, D and E requires a car. The distance in miles between the depots and the towns where the customers are is given in the following distance matrix :

| | a | b | c | d | e |
|---|---|---|---|---|---|
| A | 160 | 130 | 175 | 190 | 200 |
| B | 135 | 120 | 130 | 160 | 175 |
| C | 140 | 110 | 155 | 170 | 185 |
| D | 50 | 50 | 80 | 80 | 110 |
| E | 55 | 35 | 70 | 80 | 105 |

How should the cars be assigned to the customers so as to minimize the distance travelled ?

15. Write a short note on Travelling Salesman problem.

16. Solve the following 2×2 game graphically :

**Player B**

| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|---|
| **Player A** | $A_1$ | 2 | 1 | 0 | –2 |
| | $A_2$ | 1 | 0 | 3 | 2 |

## UNIT—V

17. Solve the following mixed - integer programming problem, using Gomory's Cutting plane method :

Maximize $Z = x_1 + x_2$

subject to the constraints

$3x_1 + 2x_2 \leq 5,$

$x_2 \leq 2,$

$x_1, x_2 \geq 0$ and $x_1$ an integer.

18. Use branch and bound method to solve the following IPP :

Maximize $Z = 2x_1 + 3x_2$

subject to the constraints

$5x_1 + 7x_2 \leq 35,$

$4x_1 + 9x_2 \leq 36,$

$x_1, x_2 \geq 0$ and are integers.

19. Use dynamic programming to show that :

$Z = p_1 \log p_1 + p_2 \log p_2 + .... + p_n \log p_n$

subject to the constraints

$p_1 + p_2 + ..... + p_n = 1$ and $pj \geq 0$

is minimum when $p_1 = p_2 = p_n = 1$.

20. Use dynamic programming to solve the LPP :

Maximize $Z = x_1 + 9x_2$

subject to the constraints

$2x_1 + x_2 \leq 25,$

$x_2 \leq 11,$

$x_1, x_2 \geq 0.$